Lawrence Hamilton June 5, 2014

PAN-ARCTIC OUTLOOK

1. Extent Projection

A Gompertz (asymmetric S curve) model estimated by iterative least squares, looking one year ahead, suggests a mean September 2014 ice extent of **4.1 million km²**. Past variations suggest a 95% confidence interval for this prediction ranging from 3.1 to 5.1 million km² (±1.0).

2. Methods / Techniques

Figure 1 shows the naive, purely statistical model. It predicts September mean extent from a Gompertz curve representing the trend over previous years. Estimation data are the NSIDC monthly mean extent reports from September 1979 through September 2013. Thus, the September 2014 extent prediction is calculated from data available in October 2013, one year in advance.

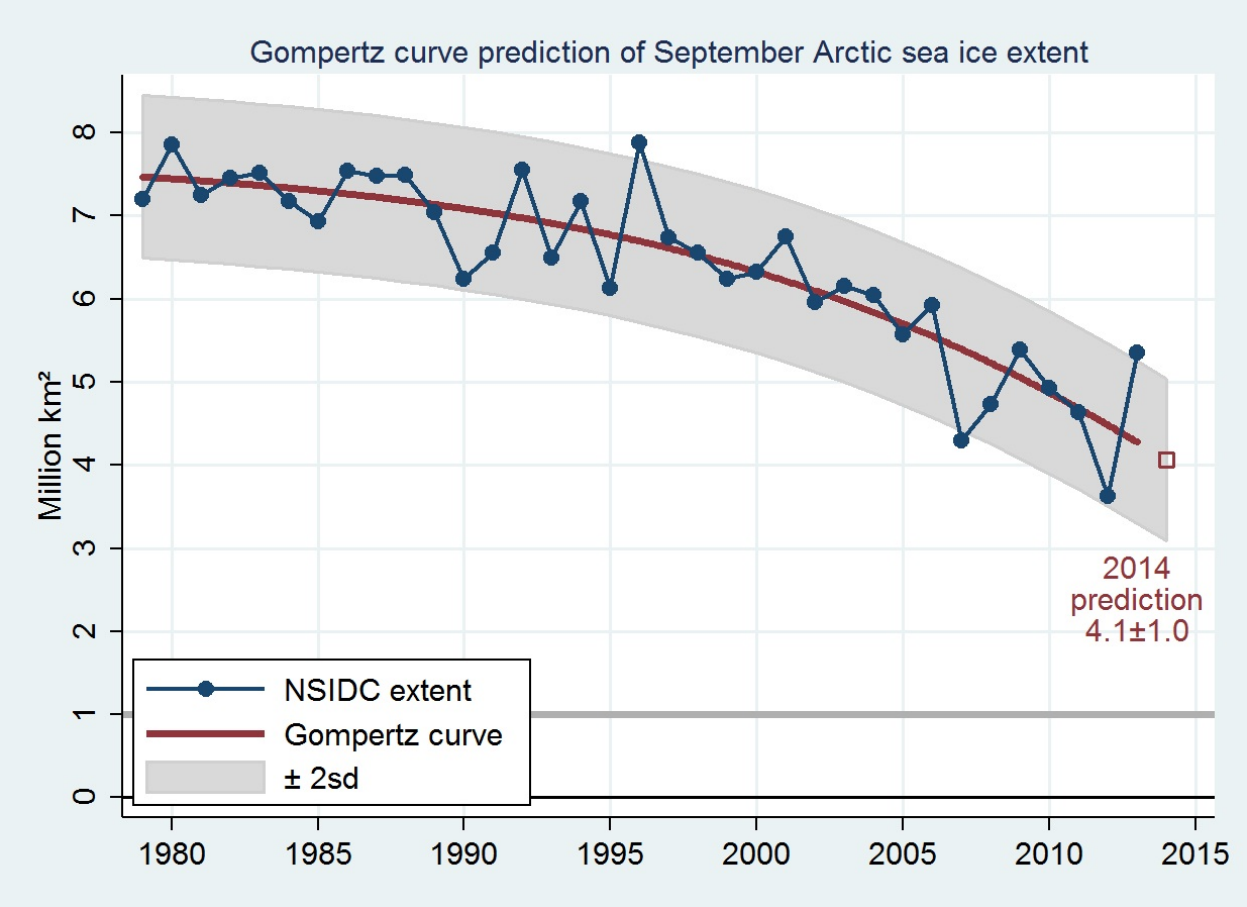

Figure 1

Parameters for the model are estimated via iterative least squares using the **nl** procedure of Stata (Hamilton 2013). Figure 1 also shows confidence bands calculated as the prediction plus or minus twice the standard deviation of the residuals.

In the command below, **gom3** specifies a 3-parameter Gompertz curve. ***extent*** refers to September mean NSIDC sea ice extent, in millions of km^2. ***year*** refers to the calendar year.

```
. nl gom3: extent year, nolog
(obs = 35)

      Source |       SS       df       MS                      Number of obs =        35
-------------+------------------------------                   R-squared     =    0.9945
       Model |   1466.5558     3  488.851933                    Adj R-squared =    0.9940
    Residual |  8.11369185    32   .25355287                    Root MSE      =  .5035403
-------------+------------------------------                   Res. dev.     =  48.16287
       Total |  1474.66949    35  42.1334141

3-parameter Gompertz function, extent = b1*exp(-exp(-b2*(year - b3)))
------------------------------------------------------------------------------
      extent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         /b1 |   7.738985    .395231    19.58   0.000     6.933926    8.544044
         /b2 |   -.082956   .0240306    -3.45   0.002    -.1319048   -.0340072
         /b3 |   2019.299   2.174724   928.53   0.000      2014.87    2023.729
------------------------------------------------------------------------------
```

In this model the first parameter, b1 = 7.74, gives the asymptotic starting level, 7.74 million km^2. The third parameter, b3 = 2019.3, gives the inflection point where this curve shifts from a steepening rate of decline to a slowing rate of decline: during the year 2019. The second parameter, b2 = –.08, controls the rate of change in the decline.

There is no significant autocorrelation ($p > .10$) among the residuals, as tested by Ljung–Box Q statistics.

```
. predict resid, resid
. corrgram resid, lag(6)
                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1       -0.2627  -0.3064   2.6293  0.1049        --|                --|
2       -0.0431  -0.1182   2.7022  0.2589          |                  |
3       -0.1774  -0.2841    3.976  0.2641         -|                --|
4        0.1318   0.0296    4.702  0.3193          |-                 |
5        0.1664   0.2453    5.897  0.3164          |-                 |-
6       -0.3260  -0.4258   10.642  0.1001        --|              ---|
```

3. Rationale

This naive, curvilinear-trend model is based on data through the end of the 2013 melt season. Most trend-line analyses of Arctic sea ice have used linear, quadratic, exponential or logistic models. The Gompertz curve appears preferable to these alternatives in several respects.

- It follows the observed pattern of gradually accelerating decline in the 1970s and 80s.
- The decline later steepens at an accelerating rate, as observed since the mid-2000s.
- The asymmetrical-S shape bears a qualitative resemblance to results from much more elaborate physical models, such as those reported by the IPCC (2007).
- Extrapolated (as speculation) into the future, model predictions do not fall below zero extent. Rather they approach this physical limit asymptotically.
- Although out-of-sample extrapolation is purely speculative, it is interesting to note the suggestion of extent falling below 1 million km^2 by 2028.

A recent meta-analysis of the ensemble skill of 309 contributions to the SEARCH Sea Ice Outlook over 2008–2013 (Stroeve et al. 2014) finds that they show collective skill (median prediction near the true extent value) in years when sea ice extent falls close to its long-term downward trend. They collectively fail (median prediction distant from the true extent) in years when sea ice extent substantially departs from this trend. The Gompertz trend-based prediction given here is therefore proposed as a rough null hypothesis: what we would expect if there is nothing but a continuation of the past 25 years' curvilinear trend.

5. Estimate of Forecast Skill

Gray bands in Figure 1 show a range of plus or minus two standard deviations around the curve. That suggests a confidence interval from 3.1 to 5.1 million km^2 for the September 2014 extent prediction.

Over 1979–2013, the standard deviation of NSIDC September ice extent is 1.07 million km^2. The standard deviation of residuals from the model in Figure 1 is just 0.49 million km^2. The squared correlation between observed and predicted values is $r^2 = .79$.

Similar Gompertz models offered the following SIO predictions for mean September extent in 2011, 2012 and 2013.

	Predicted	**Observed**
2011	4.4	4.6
2012	4.3	3.6
2013	3.8	5.3
2014	4.1	__

References

Hamilton, L.C. 2013. *Statistics with Stata*, version 12. Belmont, CA: Cengage

Stroeve, J., L.C. Hamilton, C.M. Bitz, E. Blanchard-Wrigglesworth. 2014. "Predicting September sea ice: Ensemble skill of the SEARCH Sea Ice Outlook." *Geophysical Research Letters* 41:2411–2418. doi: 10.1002/2014GL059388